2.16. Stochastic processes in random media

On the Initial Value Problem for Random Wave Equations

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Let H be a Hilbert space, and let A^2 be a selfadjoint random operator. We consider the initial value problem for the random wave equations of the form: $d^2u/dt^2 + A^2u = f$, $t \in R$ with the initial conditions, i.e. $u(0) = h_1$, $(du/dt)(0) = h_2$ (L^2 sense) on a fixed probability space (Ω, F, P), where d/dt denotes a derivative with respect to t on H. When we denote a family of projections by $\{E(x)\}$, then for any Borel function $G(x)$ on R the operator $G(A^2)$ is defined as an integral of $G(x)$ with respect to $dE(x)$ over R_+, and $D(G(A^2))$ is the totality of elements g belonging to H such that $\|G(A^2)g\|^2 < \infty$ (w.p. 1). If (h_1, h_2) belongs to a suitable class of $D(A^2) \times D(A)$-valued random variables, and if $f = f(t)$ is of a class of martingales, then there exists a unique continuous solution process with values in $D(A^2)$. Moreover if such a solution process exists then it may be expressed directly in the form: $u(t) = B(At)h_1 + A^{-1}C(At)h_2 + J(A, C, f)$ with proper functions $B(\cdot)$, $C(\cdot)$ given, where J is an integral with respect to dt the integrand of which is completely determined by A, $C(\cdot)$, $f(t)$. Under the existence and uniqueness theorem for such a Cauchy problem, one can discuss asymptotic behavior of solutions to a certain class of random wave equations.

Stochastic Processes in Random Media

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We present a small survey of stochastic processes in random media or random environments. Since such processes have a double random structure, their analysis concludes many phenomena different from classical results.

In the one-dimensional case, random walks in random media were studied fruitfully by many mathematicians. The transient process was studied by Kesten-Spitzer who obtained complete detail about the asymptotic behavior. The recurrent process was studied by Sinai who established the astonishing result that the process X_n does not behave with the usual order $n^{1/2}$ but with $\log^2 n$ in probability.

Also, in the one-dimensional case, the continuous time process (birth and death process) was studied by Golosov and Kawazu-Kesten in symmetric random environments. Golosov obtained the local limit theorem and Kawazu-Kesten obtained the global limit theorem.

A one-dimensional diffusion model in random media was studied by Brox who obtained a result analogous to Sinai's.

In the higher dimensional case, we have a few interesting works.